

SALINE AREA SCHOOLS COURSE OUTCOMES

HONORS TRIGONOMETRY

The Course

Honors Trigonometry is a 1-credit course (one semester in the high school) that serves as the fifth in a sequence of college preparatory mathematics courses at Saline High School. The key content for this fifth course is the study of the transcendental functions: exponential, logarithmic and trigonometric. Graphing calculators will be used extensively to produce and study the graphs of these functions. The key content for this course is identical to Trigonometry, however more emphasis will be placed on the rigorous derivation and proof needed for a solid theoretical foundation for further study of mathematics. In this way, the course will build upon the algebraic skills developed in Honors Algebra III, enhancing and refining these skills in such a way as to allow students to deal successfully with increasingly complex problems.

Prerequisites

Before studying Trigonometry, all students must be competent in advanced algebraic concepts, including evaluating and graphing functions and finding their domains, ranges and zeros. Students should be familiar with the transformations of functions and the inverse of a function. The prerequisites for Honors Trigonometry can be satisfied by successfully completing Honors Algebra III.

Philosophy

In Honors Trigonometry, broad concepts and widely applicable methods are emphasized. The focus of the course is neither manipulation nor memorization of an extensive taxonomy of computational methods, properties, theorems, or problem types. Although facility with manipulation and computational competence are important outcomes of the course, they are not the core of the course. Through the unifying theme of transcendental functions the course becomes a cohesive whole, rather than a collection of unrelated topics.

Symbolic reasoning and calculations with symbols are central in algebra. Through the study of algebra, a student develops an understanding of the symbolic language of the mathematics and the sciences. To this end, it is imperative that the student views algebra as more than just a game where the object is to arrive at the “right answer.” Instead, algebra must be viewed as logical process, and the student must understand that the communication of the process used to arrive at the answer is, in fact, more important than the answer itself.

The cohesive nature of the course must be emphasized through incremental development of the techniques and skills of algebra with continuous distributed review and frequent cumulative testing. Unifying themes must be revisited over and over again, with continual applications to increasingly complicated problems.

Objectives

1. Students understand the notion of an angle and how to measure it, in both degrees and radians, can convert between degrees and radians and can apply the relationship between the radius of a circle, the measure of a central angle and the length of its subtended arc.
2. Students know the definition of the *sine* and *cosine* functions as the x - and y -coordinates of points on the unit circle, are familiar with their graphs and can identify their domains and ranges. Students can also evaluate the exact values of these circular functions of multiples of $\pi/6$, $\pi/4$, $\pi/3$ and $\pi/2$.
3. Students know the definition of the *sine* and *cosine* of arbitrary angles in standard position and can also evaluate the exact values of these trigonometric functions of multiples of 30° , 45° , 60° and 90° .
4. Students know the definitions of the *secant*, *cosecant*, *tangent* and *cotangent* functions, can produce their graphs and name their domains and ranges.
5. Students can graph sinusoidal functions of the form $y = A \sin(kx + c)$ and $y = A \cos(kx + c)$ and interpret A , k and c in terms of amplitude, period and phase shift. Students use sinusoids to model the periodic behavior of oscillating quantities.
6. Students know basic trigonometric identities, including the Pythagorean identities, cofunction identities, sum and difference formulas, twice and half angle formulas, and can use these identities to prove or simplify other trigonometric identities.
7. Students can define the inverse trigonometric functions, produce their graphs and identify their domains and ranges. Students can evaluate the values of inverse trigonometric functions for certain special angles and by using a calculator and can solve trigonometric equations over limited domains and over the set of all real numbers.
8. Students can solve for missing angles and sides in right triangles using *sine*, *cosine* and *tangent* and can solve for missing angles and sides in oblique triangles using the laws of sines and cosines. Students can also find the area of a triangle given two sides and the included angle and can solve a variety of application problems involving triangles using trigonometry.
9. Students interpret the meaning of exponents, including negative, rational and real exponents, and can simplify exponential expressions using the laws of exponents. Students can graph exponential functions and use exponential functions to model both growth and decay in a variety of contexts.
10. Students can evaluate logarithms, define a logarithmic function in terms of the inverse of an exponential function and identify both its domain and range. Students can apply the properties of logarithms to simplify expressions, translate between logarithmic and exponential forms of an equation and solve exponential equations using logarithms.

Topical Outline

The outline of topics is intended to indicate the scope of the course, but not necessarily the order in which topics are taught. Although the final exam for Honors Trigonometry will be based on the topics listed in this topical outline, teachers may enrich the course with additional topics.

I. Right Triangle Trigonometry

- Prove the Pythagorean theorem and use it to find unknown sides in a right triangle.
- Derive the distance formula, find the distance between two points in the Cartesian plane, the length of a space diagonal, and the midpoint of a line segment in the Cartesian plane.
- Solve for unknown sides in 30° - 60° - 90° triangles and 45° - 45° - 90° triangles.
- Compute the sine, cosine and tangent of an acute angle in a right triangle.
- Solve for unknown sides and angles in a right triangle using sine, cosine and tangent.
- Solve a variety of problems involving right triangles.

II. Area and Volume

- Find the area of a plane figure and specify its units.
- Find the surface area and volume of a solid and specify their units.
- Use comparison of lengths and areas to compute probabilities geometrically.
- Use right triangle trigonometry to find lengths needed to compute the area and volumes of plane and solid figures.

III. Angles and Their Measures

- Measure angles using degrees-minutes-seconds and convert between degrees-minutes-seconds and decimal degrees.
- Define the radian measure of angle and convert the measure of an angle between degrees and radians.
- Apply the relationship between the radius of a circle, the measure of a central angle and the length of the subtended arc and solve problems involving circular motion.
- Solve problems involving rotational velocity.
- Sketch angles given in both degrees and radians in standard position.

IV. Circular Functions

- Define the sine and cosine functions in terms of the x - and y -coordinates of a point on the unit circle.
- Define the secant, cosecant, tangent and cotangent functions in terms of sine and cosine.
- Evaluate the circular functions of multiples of $\pi/6$, $\pi/4$, $\pi/3$ and $\pi/2$.
- Evaluate the circular functions on a calculator.
- Prove the Pythagorean identity for sine and cosine and use it to derive the Pythagorean identities for secant and tangent and cosecant and cotangent.

- Graph the circular functions, identify the functions that are even and the functions that are odd and identify their symmetries and identify the period of each function.
- Graph sinusoidal functions of the form $y = A \sin(kx + c)$ and $y = A \cos(kx + c)$ and interpret A , k and c in terms of amplitude, period and phase shift.
- Use sinusoids to model the periodic behavior of oscillating quantities such as voltage and sound waves.

V. Trigonometric Functions of Angles

- Define the sine and cosine of an arbitrary angle in standard position..
- Evaluate the trigonometric functions of multiples of 30° , 45° , 60° and 90° using reference angles.
- Evaluate the trigonometric functions of an angle on a calculator.
- Use trigonometric functions to find the magnitude and direction and components of a two-dimensional vector.

VI. Inverse Trigonometric Functions

- Define arcsine, arccosine and arctangent as the inverse of the sine, cosine and tangent functions, respectively.
- Graph the inverse trigonometric functions and identify their domains and ranges.
- Evaluate the inverse trigonometric functions for multiples of $\pi/6$, $\pi/4$, $\pi/3$ and $\pi/2$
- Evaluate the inverse trigonometric functions on a calculator.

VII. Trigonometric Identities, Formulas and Equations

- Know and use the reciprocal, quotient, Pythagorean, and cofunction identities.
- Derive and use the sum and difference formulas for sine, cosine and tangent.
- Derive and use the twice and half angle formulas for sine, cosine and tangent.
- Derive and use the power reduction formulas for sine and cosine.
- Simplify trigonometric expressions using the basic identities.
- Verify trigonometric identities both graphically and analytically.
- Solve trigonometric equations over the set of all real numbers and over a limited domain.

VIII. Oblique Triangles

- Prove the law of sines and use it to solve for unknown sides and angles in oblique triangles.
- Prove the law of cosines and use it to solve for unknown sides and angles in oblique triangles.
- Find the area of a triangle given two sides and the included angle.
- Derive Heron's formula and use it to find the area of a triangle.
- Solve a variety of problems involving oblique triangles, including problems in surveying.

IX. Exponential Functions

- Know and apply the laws of exponents for positive, zero and negative exponents.
- Know and apply the properties of radicals to simplify radical expressions.
- Interpret rational exponents in terms of radicals and simplify expressions containing rational exponents.
- Graph exponential functions, identify their domains and ranges, and interpret irrational exponents in terms of the limit of an exponential function.
- Define Euler's number, e , graph the natural exponential function and use it to model problems of both growth and decay, including population problems and continuously compounded interest problems.

X. Logarithmic Functions

- Find the inverse of a function and interpret the inverse of a function graphically.
- Define logarithmic functions, including the common logarithm and the natural logarithm, as the inverse of exponential functions, sketch their graphs and identify their domains and ranges.
- Convert between logarithmic and exponential forms of an equation and evaluate logarithms.
- Use the product property, quotient property, power property, inverse properties and change of bases properties of logarithms to simplify logarithmic expressions.

XI. Exponential and Logarithmic Equations

- Solve exponential equations using common bases, common logarithms and natural logarithms.
- Solve logarithmic equations.
- Use exponential equations to solve compound interest problems.
- Use exponential equations to solve problems of exponential growth and decay, including population growth and radioactive decay.