

SALINE AREA SCHOOLS

COURSE OUTCOMES

HONORS GEOMETRY

The Course

Honors Geometry is a 1-credit course (one semester in the high school block) that serves as the third in a sequence of college preparatory mathematics courses. The key content for this third course includes the measurement of lengths, angles, areas and volumes and the geometry of lines, polygons and circles. Students should also become familiar with formal logic and mathematical proof, including both inductive and deductive reasoning.

Prerequisites

Before studying geometry, all students must be competent in basic algebra, including solving both linear and quadratic equations and systems of linear equations. Students should also be competent in evaluating roots of real numbers and simplifying radical expressions.

The prerequisites for Honors Geometry can be satisfied by successfully completing Algebra II.

Philosophy

In Honors Geometry, broad concepts and widely applicable methods are emphasized. The focus of the course is neither manipulation nor memorization of an extensive taxonomy of computational methods, properties, theorems, or problem types. Although facility with manipulation and computational competence are important outcomes of the course, they are not the core of the course. Through the unifying themes of reasoning, measurement, and application, the course becomes a cohesive whole, rather than a collection of unrelated topics. These themes are developed using lines, segments, angles, polygons, circles and solids.

Formal logic and proof are central in all mathematics but are traditionally introduced in the geometry course, leading to the common misconception that proof is only necessary in geometry and must be accomplished in the common two-column format. The goal, however, is to lay the groundwork for logical argument throughout the students mathematical career. To this end, it is imperative that the student begins to view mathematics as an axiomatic science, one built on postulates (which are assumptions), definitions (which are choices) and theorems (which follow logically from the postulates and definitions). Thus, it is necessary to introduce the student to different forms of logical reasoning (both inductive and deductive), different forms of proof (both direct and indirect) and different formats for writing proofs (both two-column and paragraph), while continually emphasizing the *reason* for doing proofs.

The cohesive nature of the course must be emphasized through incremental development of the techniques and skills of geometry with continuous distributed review and frequent cumulative testing. Unifying themes must be revisited over and over again, with continual applications to increasingly complicated problems.

Objectives

1. Students generate and analyze patterns found in geometric situations. They draw diagrams, gather data, make conjectures, judge the validity of a logical argument and give counterexamples to disprove a statement.
2. Students are able to distinguish between postulates, definitions and theorems and can give examples of each. They write formal geometric proofs in both two-column and paragraph format, including proof by contradiction.
3. Students can measure the length of a line segment using a ruler, can measure an angle using a protractor and can construct line segments, angles and bisectors using a compass and a straight edge.
4. Students compute the perimeter and area of plane figures, the circumference and area of circles, and the volume and surface areas of solids. Students use comparison of lengths and areas to compute probabilities geometrically.
5. Students know the effect of rigid and non-rigid transformations on figures in the coordinate plane, including rotations, translations, reflections and dilations.
6. Students use the relationships between complementary, supplementary, vertical and exterior angles and use them to solve problems and prove theorems, including theorems involving parallel lines, perpendicular lines and parallel lines cut by a transversal.
7. Students classify polygons by their number of sides and know the properties of regular polygons. Students classify special quadrilaterals and know their properties and can prove theorems involving quadrilaterals.
8. Students prove theorems involving the congruence and similarity of triangles and use the concept of corresponding parts of both congruent and similar figures.
9. Students can classify triangles based on the measure of their angles and the lengths of their sides and can apply the isosceles triangle theorem and the triangle inequality theorem. They can find the lengths of the sides in a right triangle using the Pythagorean theorem and know the relationship between the sides and angles of both 30° - 60° - 90° and 45° - 45° - 90° triangles.
10. Students can identify the radius and diameter of a circle and can compute the measures of central angles, inscribed angles, intercepted arcs and the area of sectors.

11. Students use algebra to solve problems in geometry, including solving linear equations and systems of linear equations and solving quadratic equations by factoring and by the quadratic formula.
12. Students can apply geometric concepts to solve problems in a variety of contexts, including construction, surveying, navigation and scale modeling.

Topical Outline

The outline of topics is intended to indicate the scope of the course, but not necessarily the order in which topics are taught. Although the Final Exam for Geometry will be based on the topics listed in this topical outline, teachers may enrich the course with additional topics.

I. Postulates, Definitions, Theorems and Proof

Postulates, Definitions and Theorems

- Understand the difference between an axiom, a definition and a theorem.
- Know the postulates of Euclidean geometry involving points, lines and planes.

Geometric Intuition and Inductive Reasoning

- Draw diagrams, collect data and use patterns to make conjectures.
- Disprove an assertion by finding one counterexample.
- Develop, use and apply Pascal's Triangle and Fibonacci sequences.

Deductive Reasoning and Proof

- Distinguish between deductive and inductive reasoning.
- Use properties from algebra to construct two-column algebraic proofs.
- Write direct geometric proofs in both two-column and paragraph format.
- Understand the relationship between a conditional statement, its converse, inverse and contrapositive, and know which are logically consistent.
- Write indirect geometric proofs in both two-column and paragraph format.
- Distinguish between valid and invalid arguments and identify flaws in an invalid argument.
- Use logic symbols and notation.

II. Measurement in Geometry

Measurement

- Measure the length of a line segment using a ruler in both metric (decimal centimeters) and English (fractions of an inch) units.
- Find the distance between two points and the midpoint of a line segment in the coordinate plane.
- Find the measure of an angle in degrees using a protractor.
- Categorize angles as acute, right, obtuse or straight based on their angle measure.

Constructions

- Construct congruent segments and congruent angles using a compass and a straight edge.
- Bisect line segments and angles using a compass and a straight edge.

Perimeter and Area

- Find the perimeter of a polygon and specify its units..
- Find the area of a rectangular region and specify its units.
- Find the volume of a right parallelepiped and specify its units.
- Use comparison of lengths and areas to compute probabilities geometrically.

III. Congruency and Similarity

Congruent and Similar Figures

- Define congruent and similar figures and identify figures that are congruent and similar.
- Use the concepts of ratio and proportion to find lengths in similar figures.
- Compute the geometric mean of two numbers.
- Use similarity to solve problems involving scale drawings and scale models.
- Understand and apply the relationship between scale factors of similar figures and their perimeters, areas and volumes.

Transformations

- Identify and sketch rigid transformations in a plane, including reflections, translations and rotations.
- Identify and sketch nonrigid transformations (dilations) in a plane.
- Determine if a figure has rotational and reflectional symmetry.

IV. Points, Lines, Planes and Angles

- Categorize lines as parallel, perpendicular, oblique and skew.
- Categorize angles formed by lines as supplementary, complementary, right and vertical and know their properties.
- Categorize angles formed by parallel lines cut by a transversal as interior, exterior, alternate, same side and corresponding and know their properties.
- Prove theorems involving lines and angles formed by lines.

V. Triangles

Classifying Triangles

- Classify triangles as scalene, isosceles and equilateral.
- Classify triangles as acute, right and obtuse.
- Know the definitions and properties of medians, altitudes and angle bisectors in triangles.
- Find the measure of interior and exterior angles.
- Know and apply the triangle inequality theorem.

Congruent Triangles

- Prove triangles congruent using SSS, SAS, ASA, AAS and HL.
- Use corresponding parts of congruent triangles to prove theorems.

Similar Triangles

- Prove triangles similar using AA~, SSS~, and SAS~ .
- Use corresponding parts of similar triangles to solve problems and prove theorems.

Area of a Triangle

- Find the area of a triangle given its height and the length of its base.

Right Triangles and the Pythagorean Theorem

- Prove the Pythagorean theorem and use it to find unknown sides in a right triangle.
- Use the converse of the Pythagorean theorem to determine if a triangle is acute, right or obtuse.
- Solve for unknown sides in 30° - 60° - 90° triangles and 45° - 45° - 90° triangles.

VI. Polygons

Regular Polygons

- Name polygons according to their number of sides and classify them as convex, concave and regular.
- Find the measure of interior and exterior angles of a regular polygon.
- Use patterns to predict the number of diagonals in a polygon.

Quadrilaterals

- Classify quadrilaterals as parallelograms, kites and trapezoids and know the properties of each.
- Define a parallelogram and know its properties.
- Prove a quadrilateral is a parallelogram.
- Classify parallelograms as rectangles, rhombuses and squares and know their properties.
- Find the area of a parallelogram given its height and the length of its base.
- Define a trapezoid and an isosceles trapezoid and know their properties.
- Find the area of a trapezoid given its height and the lengths of its bases.

VII. Circles

- Identify the radius and diameter of a circle.
- Find the circumference, area, arc length and area of a sector of a circle.
- Find the measure of central angles, inscribed angles and their intercepted arcs.

VIII. Surface Area and Volume

Prisms and Cylinders

- Find the surface area of right prisms.
- Find the volume of both right and oblique prisms.
- Find the surface area of a right cylinder.
- Find the volume of both right and oblique cylinders.

Pyramids and Cones

- Find the lateral area and total surface area of right pyramids.
- Find the volume of both right and oblique pyramids.
- Find the lateral area and total surface area of a right cone.
- Find the volume of both right and oblique cones.

Spheres

- Find the surface area of a sphere.
- Find the volume of a sphere.