SALINE AREA SCHOOLS
COURSE OUTCOMES

HONORS ALGEBRA III

The Course
Honors Algebra III is a 1-credit course (one semester in the high school) that serves as the fourth in a sequence of college preparatory mathematics courses at Saline High School. The key content for this course is identical to Algebra III, however greater emphasis will be placed on rigor and proof in this honors course than in the regular course. Additional topics not covered in Algebra III, including mathematical induction, the binomial expansion theorem, linear programming and curve fitting, will be also be explored.

Prerequisites
Before studying Honors Algebra III, all students must be competent in basic algebraic concepts, including evaluating algebraic expressions, solving and graphing linear equations and inequalities, solving polynomial equations by factoring and simplifying radical expressions. Students should also be comfortable with deductive reasoning and proof as developed in a college preparatory geometry course. The prerequisites for Honors Algebra III can be satisfied by successfully completing Honors Geometry **or** Geometry with recommendation of the instructor.

Philosophy
In Honors Algebra III, broad concepts and widely applicable methods are emphasized. The focus of the course is neither manipulation nor memorization of an extensive taxonomy of computational methods, properties, theorems, or problem types. Although facility with manipulation and computational competence are important outcomes of the course, they are not the core of the course. Through the unifying theme of patterns the course becomes a cohesive whole, rather than a collection of unrelated topics. Patterns, relationships and functions comprise one of the most important themes in the study of mathematics. Mathematical thinking begins with the recognition of similarities among objects or events, proceeds to generalization and abstraction, and culminates in the ability to understand, explain and make predictions. Contexts that exhibit structure and regularity provide rich opportunities for describing the physical world, studying mathematics and solving problems.

Symbolic reasoning and calculations with symbols are central in algebra. Through the study of algebra, a student develops an understanding of the symbolic language of the mathematics and the sciences. To this end, it is imperative that the student views algebra as more than just a game where the object is to arrive at the "right answer." Instead, algebra must be viewed as logical process, and the student must understand that the communication of the process used to arrive at the answer is, in fact, more important than the answer itself.

The cohesive nature of the course must be emphasized through incremental development of the techniques and skills of algebra with continuous distributed review and frequent cumulative testing. Unifying themes must be revisited over and over again, with continual applications to increasingly complicated problems.

Objectives

1. Students can identify patterns in a sequence of numbers by recognizing both arithmetic and geometric patterns and using finite differences. Students can find the *n*th term and means of both arithmetic and geometric sequences and compute finite arithmetic and infinite geometric series.

2. Students can organize data in an array using matrices and are familiar with the algebra of matrices. Students can represent systems of linear equations using matrices, can solve systems using Gaussian elimination with back substitution, using Cramer's rule and using matrix equations, either by hand or on the graphing calculator, and can interpret the solutions graphically.

3. Students can distinguish between a relation and a function and can identify relations and functions as well as their domains and ranges algebraically (from their formulas or equations), numerically (from lists of ordered pairs) and graphically.

4. Students understand that the graph of a function is the graph of the set of *all* ordered pairs (x,y) that *satisfy* the equation $y = f(x)$. Students should be able to graph functions by plotting points and are able to produce the graph of a function in an arbitrary viewing window on a graphing calculator.

5. Students associate the zeros of the function $f(x)$ with the roots of the equation $f(x) = 0$ and the x-intercepts of the graph of the equation $y = f(x)$ and can solve equations by finding zeros of a function on a graphing calculator.

6. Students are able to solve and graph both linear and quadratic functions and solve problems using linear and quadratic functions.

7. Students understand the fundamental theorem of algebra, can factor an *n*th degree polynomial into *n* complex linear factors and can solve polynomial equations by factoring.

8. Students appreciate mathematics as a system of logic based on axioms, postulates, definitions and theorems (properties). To this end, students are able to *prove* the existence of a solution to a linear or matrix equation by *justifying* each step in the solution process with an axiom, postulate, definition or theorem (property). Although it is impractical to demand such detail each time an equation is solved, students must appreciate the need for and be proficient at showing the steps that lead to the solution of an equation or inequality.

Topical Outline
The outline of topics is intended to indicate the scope of the course, but not necessarily the order in which topics are taught. Although the final exam for Honors Algebra III will be based on the topics listed in this topical outline, teachers may enrich the course with additional topics.

I. Sequences and Series

- Use patterns such as finite differences and recursion formulas to identify the *n*th term in a sequence of numbers.
- Write a series using summation notation.
- Identify arithmetic sequences and means, write an expression for the *n*th term in an arithmetic sequence and compute a finite arithmetic series.
- Identify geometric sequences and means, write an expression for the *n*th term in a geometric sequence and compute a finite geometric series.
- Compute an infinite geometric series.
- Recognize patterns in problems and solve them using sequences and series.
- prove theorems involving patterns using mathematical induction.
- Simplify expressions of the form $(a + b)^n$ using the binomial expansion theorem.

II. Matrices

- Use matrices to represent an array of data.
- Add matrices and identify the identity element of matrix addition.
- Identify the additive inverse (opposite) of a matrix as defined by the inverse property of matrix addition.
- Define matrix subtraction in terms of additive inverse.
- Multiply a matrix by a scalar.
- Multiply matrices and identify the identity element of matrix multiplication.
- Find the inverse of a 2 x 2 matrix.
- Solve matrix equations and justify each step in the solution process with an axiom, definition or theorem (property).
- Use graphing calculators to solve matrix equations of higher order.
- Use matrix equations to solve problems.

III. Systems of Linear Equations

- Solve a system of linear equations using the method of substitution.
- Solve a system of linear equations using the method of elimination.
- Distinguish between consistent and inconsistent systems and dependent and independent systems.
- Interpret systems of linear equations in two variables graphically.
- Use matrices to represent a system of linear equations.
- Solve a system of linear equations using Gaussian elimination with back substitution.
- Compute the determinant of 2 x 2 and 3 x 3 matrices and understand the consequences of the matrix being singular.
- Solve a 2 x 2 system of linear equations by Cramer's rule.
- Solve a 3 x 3 system of linear equations by Cramer's rule using the graphing calculator.

IV. Relations and Functions

- Find the distance between two points and the midpoint of a line segment in the Cartesian Coordinate System.
- Understand that a relation is any well-defined set of ordered pairs (x,y) and that the graph of a relation is the graph of all its ordered pairs.
- Draw a mapping diagram of a relation and identify its domain and range.
- Write the equation of a circle in standard form and sketch its graph.
- Write the equation of an ellipse in standard form and sketch its graph.
- Identify relations that are functions using the vertical line test.
- Produce the graph of a function on the graphing calculator.
- Evaluate functions using function notation.
- Find the zeros of a function.
- Find the implied domain of a function.
- Identify odd and even functions and their symmetries.
- Produce the graphs of functions by translating, reflecting and stretching graphs of familiar functions. Specifically, sketch the graphs of $y = f(x - h)$, $y = f(x) + k$, $y = -f(x)$, $y = f(-x)$, $y = k\,f(x)$, $y = f(kx)$, $y = |f(x)|$ and $y = f(|x|)$.
- Produce new functions by the combination and composition of familiar functions.
- Find the inverse of a function and interpret inverses graphically.
- Use functions to model and solve problems.

V. Linear Functions

- Solve linear equations numerically and identify roots of a linear equation with zeros of a linear function and the x-intercept of its graph.
- Solve linear equations in one variable.
- Solve linear inequalities in one variable.
- Compute the slope of a line given two points on the line.
- Graph a linear equation using slope-intercept form.
- Write the equation of a line using point-slope form.
- Write the equation of a best-fit line through a set of data.
- Use linear equations and functions to solve problems.
- Use linear programming to solve problems.

VI. Quadratic Functions

- Solve quadratic equations numerically and identify roots of a quadratic equation with zeros of a quadratic function and the x-intercepts of its graph.
- Graph quadratic equations in the form $y = a(x - h)^2 + k$ and explain how the graph of the parabola changes as a, h, and k vary.
- Solve quadratic equations by factoring and by extracting square roots.
- Define the imaginary number i, simplify expressions involving imaginary and complex numbers and apply the complex conjugates theorem.

- Solve quadratic equations by completing the square.
- Derive the quadratic formula and use it to solve quadratic equations.
- Solve quadratic inequalities by graphing and by sign charts, including quadratics with irrational roots.
- Use quadratic equations and functions to solve problems.
- Fit a curve to data.

VII. Polynomial Functions

- Solve polynomial equations numerically and identify roots of a polynomial equation with zeros of a polynomial function and the *x*-intercepts of its graph.
- Divide polynomial functions using long division and synthetic division.
- Use the intermediate value theorem to identify intervals between integers containing real roots of a polynomial function.
- Factor polynomials according to the fundamental theorem of algebra using DesCartes' rule of signs, the rational roots theorem, the remainder theorem, synthetic division, the factor theorem, the complex conjugates theorem and upper and lower bounds.
- Write the equation of the simplest polynomial function having given roots.
- Solve polynomial equations by factoring.
- Solve polynomial inequalities by graphing and by sign charts.
- Use polynomial equations and functions to solve problems.

VIII. Rational Functions (*optional*)

- Solve rational equations numerically and identify roots of a rational equation with zeros of a rational function and the *x*-intercepts of its graph.
- Identify vertical asymptotes and holes of the graphs of rational functions.
- Identify horizontal asymptotes and other endpoint behaviors of the graphs of rational functions.
- Solve rational equations analytically and identify extraneous roots.
- Solve rational inequalities by graphing and by sign charts.
- Use rational equations and functions to solve problems, including problems of direct and inverse variation.